

# Risk-Neutral Skewness: Evidence from Stock Options

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## Abstract

We investigate the relative importance of various factors in explaining the volatility skew observed in the prices of stock options traded on the Chicago Board Options Exchange. The skewness of the risk-neutral density implied by individual stock option prices tends to be more negative for stocks that have larger betas, suggesting that market risk is important in pricing individual stock options. Also, implied skewness tends to be more negative in periods of high market volatility, and when the risk-neutral density for index options is more negatively skewed. Other firm-specific factors, including firm size and trading volume also help explain cross-sectional variation in skewness. However, we find no robust relationship between skewness and the firm's leverage. Nor do we find evidence that skewness is related to the put/call ratio, which may be viewed as a proxy for trading pressure or market sentiment. Overall, firm-specific factors seem to be more important than systematic factors in explaining the variation in the skew for individual firms.

## I. Introduction

Under the assumptions of the risk-neutral pricing paradigm, the price of an option should be equal to its discounted expected payoff under the risk-neutral measure. This expectation may be calculated by integrating the payoff function over a risk-neutral density function. The problem is in knowing what risk-neutral density to use. Under Black-Scholes, the risk-neutral density is lognormal, but this prediction has been convincingly rejected (see, for example, MacBeth and Merville (1979) and Rubinstein (1985)). In response, academic research has proceeded in two directions. One has been to specify alternative stochastic processes, which in turn imply alternative risk-neutral densities. The other has been to develop procedures for backing out implied risk-neutral density functions from observed option prices (see Rubinstein (1994) and the survey paper by Jackwerth (1999)). This literature has found that implied risk-neutral densities tend to be

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more negatively skewed than the lognormal density, and that the amount of skewness varies over time.

In this paper, we investigate the cross-sectional and time-series determinants of risk-neutral skews implicit in the prices of individual stock options. Using the skewness metric of Bakshi, Kapadia, and Madan (BKM, hereafter) (2000), we test whether leverage, firm size, beta, trading volume, and/or the put/call volume ratio can explain cross-sectional variation in risk-neutral skew. We also test whether the systematic risk-neutral skewness or market volatility reflected in index options helps explain time-series variation in the skewness of individual stock options. Our hope is that a better understanding of the sources of risk-neutral skew will help guide us in the future as we work to develop better option models.

Our analysis begins with trade and quote data for options listed on the Chicago Board Options Exchange, as reported in the Berkeley Options Data Base. The data set covers a period of more than 10 years, from April 7, 1986, through December 31, 1996, and includes options of all strikes and maturities for 1,421 underlying stocks and the S&P 500 index. Put/call volume ratios were also calculated from the Berkeley Options Data Base, and additional data on firm characteristics were extracted from CRSP and COMPUSTAT.

The procedure in BKM (2000) was applied to obtain weekly estimates of risk-neutral skewness for each underlying stock. BKM (2000) show that the moments of the risk-neutral density can be expressed in terms of the prices of payoffs that depend on the future stock price. Bakshi and Madan (2000) show that any payoff function with bounded expectations can be spanned by a continuum of out-of-the-money calls and puts. Therefore, the prices of these payoffs can be expressed as a linear combination of the prices of the calls and puts. This provides a simple way to compute the risk-neutral skewness.

Cross-sectional analysis was then performed to determine how risk-neutral skewness is related to leverage, firm size, beta, trading volume, and the put/call volume ratio. Contrary to results reported by Toft and Prucyk (1997), we find that firms with more leverage have less negative skews. We also find that firms with larger betas and larger market values have more negative skews while firms with higher trading volume have more positive skews. However, we do not find a robust cross-sectional relationship between the risk-neutral skew and the put/call volume ratio.

In addition to the cross-sectional analysis, we also determine the extent to which individual stock skews are influenced by market volatility and market skewness by estimating a pooled cross-sectional time-series regression. We find that individual stock skews tend to be more negative (skewed left) when market volatility is higher and when the market skew is more negative.

The remainder of this paper is organized as follows. In Section II, we describe the construction of our measure for the risk-neutral skew. Section III then describes the motivation for and construction of the independent variables. The sample properties are discussed in Section IV, followed by our main regression results in Section V and robustness analysis in Section VI. Section VII discusses other results relating leverage to the skew and Section VIII contains our concluding remarks.

## II. Measuring Risk-Neutral Skewness

There has not been much empirical work on the skewness characteristics of individual stock options. Rubinstein (1985) studied the skew using two years of tick data (August 1976 through August 1978) for options on 30 stocks. Comparing implied volatilities on carefully selected pairs of options, he found statistically significant violations of the Black-Scholes (1993) model. Rubinstein's most intriguing result was that the direction of the bias changed signs between subperiods, implying that the skewness of the risk-neutral density changed over time. However, Rubinstein did not analyze the cross-sectional or time-series determinants of the skew.

One effort to measure and explain cross-sectional differences in skews across stocks is a study by Toft and Prucyk (1997). They propose a skewness metric that is proportional to the slope of the implied volatility curve (or "smile") divided by the implied volatility of an at-the-money option. Because their metric impounds information in both the level and the slope, it is unclear exactly how to interpret their result. Is it that more highly levered firms have more downward-sloping implied volatility curves or lower implied volatilities? We provide a brief discussion of their analysis later in the paper.

Although we study the skew of the risk-neutral density rather than the slope of the implied volatility curve, there is a one-to-one mapping between the risk-neutral density function and the implied volatility curve. Negatively sloped volatility curves, where the implied volatility of out-of-the-money puts is higher than that of in-the-money puts, correspond to negative skewness in the risk-neutral density. In their paper, BKM (2000) verify a high correlation between their measure of the skewness and the slope of the implied volatility curve. In the remainder of this section, we briefly describe the method used to construct the skewness measure from option prices. We also discuss some biases that are introduced into the measure when using option price data that do not contain a continuum of strike prices.

The risk-neutral density is the probability density function for the future stock price that prices a cross section of calls and/or puts with different strike prices. Specifically, let  $S(t)$  be the stock price at time  $t$  and  $f(S(t + \tau))$  be the risk-neutral density for the stock price at time  $t + \tau$ . Then the risk-neutral valuation equation for calls is

$$\begin{aligned} C(t, \tau; K) &= e^{-r\tau} E_t^* [\max(S(t + \tau) - K, 0)] \\ &= e^{-r\tau} \int_{-\infty}^{\infty} \max(S(t + \tau) - K, 0) f(S(t + \tau)) dS(t + \tau), \end{aligned}$$

where  $C(t, \tau; K)$  is the price of the call option at time  $t$  with strike price  $K$  and  $\tau$  years to maturity,  $r$  is the annual risk-free rate, and  $E^*$  is the expectation taken with respect to the equivalent risk-neutral measure.

Many techniques have been suggested for estimating risk-neutral densities. For example, one may use the fact that the risk-neutral density is the second derivative of the option price with respect to the strike price, as in Breeden and Litzenberger (1978) and Shimko (1993). Rubinstein (1994) and others have suggested nonparametric approaches. As we are interested only in the skewness of

the risk-neutral density, not the entire density, these methods are unnecessarily cumbersome. We use a more direct approach to measuring the risk-neutral skew, based on BKM (2000). This measure of the skewness is easy to compute and has the advantage of not relying on any particular pricing model.

The basic idea is that any payoff function can be spanned by a continuum of out-of-the-money calls and puts (Bakshi and Madan (2000)). In particular, the risk-neutral skewness can be expressed as a function of the current price of three payoffs: a quadratic, cubic, and quartic payoff, where the payoffs are defined as the stock's continuously compounded return taken to the second, third, and fourth power, respectively. The quadratic, cubic, and quartic payoffs can, in turn, be expressed as a linear combination of current out-of-the-money option prices. Thus, risk-neutral skewness can be expressed as a function of current option prices.

The skew is computed as follows. Let the  $\tau$  period continuously compounded return on the underlying asset,  $S$ , be  $R(t, \tau) \equiv \ln[S(t + \tau)/S(t)]$ . Let the operator  $E^*$  represent the expectation under the equivalent risk-neutral measure. The time  $t$  price of a quadratic, cubic, and quartic payoff received at time  $t + \tau$  can then be expressed as  $V(t, \tau) \equiv E_t^*\{e^{-r\tau}R(t, \tau)^2\}$ ,  $W(t, \tau) \equiv E_t^*\{e^{-r\tau}R(t, \tau)^3\}$ ,  $X(t, \tau) \equiv E_t^*\{e^{-r\tau}R(t, \tau)^4\}$ , respectively. By Theorem 1 in BKM (2000), we have that the  $\tau$  period risk-neutral skewness is

$$(1) \quad \text{SKEW}(t, \tau) \equiv \frac{E_t^* \left\{ (R(t, \tau) - E_t^*[R(t, \tau)])^3 \right\}}{\left\{ E_t^* (R(t, \tau) - E_t^*[R(t, \tau)])^2 \right\}^{3/2}} \\ = \frac{e^{r\tau} W(t, \tau) - 3\mu(t, \tau)e^{r\tau} V(t, \tau) + 2\mu(t, \tau)^3}{[e^{r\tau} V(t, \tau) - \mu(t, \tau)^2]^{3/2}}.$$

The time  $t$  prices of the time  $t + \tau$  volatility, cubic, and quartic payoffs are given as a weighted sum of out-of-the-money (OTM) calls and puts,

$$(2) \quad V(t, \tau) = \int_{S(t)}^{\infty} \frac{2 \left( 1 - \ln \left[ \frac{K}{S(t)} \right] \right)}{K^2} C(t, \tau; K) dK \\ + \int_0^{S(t)} \frac{2 \left( 1 + \ln \left[ \frac{S(t)}{K} \right] \right)}{K^2} P(t, \tau; K) dK, \\ (3) \quad W(t, \tau) = \int_{S(t)}^{\infty} \frac{6 \ln \left[ \frac{K}{S(t)} \right] - 3 \left( \ln \left[ \frac{K}{S(t)} \right] \right)^2}{K^2} C(t, \tau; K) dK \\ - \int_0^{S(t)} \frac{6 \ln \left[ \frac{S(t)}{K} \right] + 3 \left( \ln \left[ \frac{S(t)}{K} \right] \right)^2}{K^2} P(t, \tau; K) dK,$$

$$(4) \quad X(t, \tau) = \int_{S(t)}^{\infty} \frac{12 \left( \ln \left[ \frac{K}{S(t)} \right] \right)^2 - 4 \left( \ln \left[ \frac{K}{S(t)} \right] \right)^3}{K^2} C(t, \tau; K) dK \\ + \int_0^{S(t)} \frac{12 \left( \ln \left[ \frac{S(t)}{K} \right] \right)^2 + 4 \left( \ln \left[ \frac{S(t)}{K} \right] \right)^3}{K^2} P(t, \tau; K) dK,$$

and

$$\mu(t, \tau) = E_t^* \ln \left[ \frac{S(t + \tau)}{S(t)} \right] \\ \approx e^{r\tau} - 1 - \frac{e^{r\tau}}{2} V(t, \tau) - \frac{e^{r\tau}}{6} W(t, \tau) - \frac{e^{r\tau}}{24} X(t, \tau).$$

To empirically estimate the skewness, we need to approximate the integrals in equations (2), (3), and (4) above using observed option prices. We use a trapezoidal approximation to estimate the integrals using discrete data. Since we do not have access to a continuum of option prices, bias may be introduced into our estimate of the skew. There are two sources of this bias: the discreteness of the strike price interval and asymmetry in the domain of integration. This bias is likely to be more pronounced for individual stock options than it is for index options—compared to index options, the strike prices of stock options are fewer and more coarsely spaced. To assess the magnitude of these two biases, we perform a number of simulation experiments.

First, to assess the impact of a discrete strike price interval, we generate Black-Scholes option prices with one month to maturity using a volatility of 20% per annum, a risk-free rate of 7%, and a stock price of  $S(t) = \$50$ . Since the prices come from the Black-Scholes model, we know that the skewness should be zero. Figure 1 plots the BKM estimate of the skew vs. strike price interval from \$1 to \$5, where we integrate over strike prices from 30 to 70. A strike price interval of \$5 induces a bias of roughly  $-0.07$ , and an interval of \$2.50 induces a bias of roughly  $-0.05$ . Unfortunately, there is not much that we can do about the discrete strike price interval, since that is the nature of the data. However, since most of our data have a strike price interval of \$5 or \$2.50, the bias is roughly the same for all observations in our sample, and we should still be able to discern cross-sectional differences in the data.

Second, the domain of integration may not be symmetric. Consider the case where we have data on one OTM put but two OTM calls. Since the prices of the three payoffs,  $V(t, \tau)$ ,  $W(t, \tau)$ ,  $X(t, \tau)$ , depend on the difference between the weighted average of OTM calls and OTM puts, having more observations on calls than puts could easily introduce bias. To assess the magnitude of this bias, we keep the parameters the same as in the previous experiments, except we perform the integration over the domain  $[S(t) - Z, 100]$  where we vary  $Z$  from one to 40. The results are graphed in Figure 2. The bias that is introduced by asymmetry can be significant. For example, if we have complete observations on OTM calls

FIGURE 1  
Plot of Skew vs. Strike Price Interval

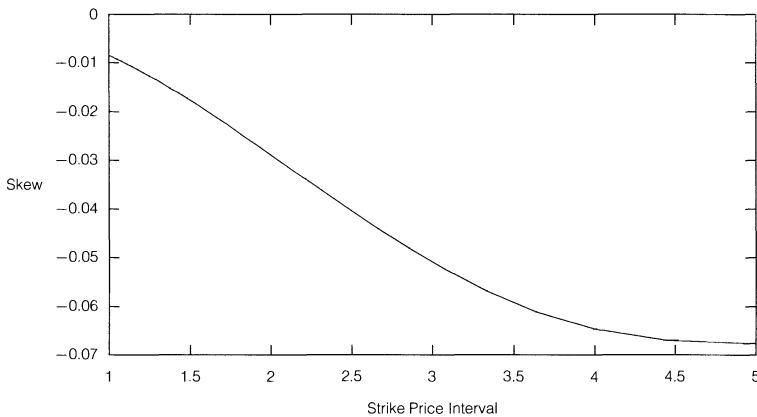


Figure 1 shows the bias that is introduced into the skew measure by a discrete strike price interval. Option prices are computed from the Black-Scholes model at evenly spaced strike price intervals from \$30 to \$70. The options have one month to maturity, the underlying stock is \$50, the volatility is 20% per annum, and the risk-free rate is 7%. The risk-neutral skew is then computed from these prices using the method outlined in Bakshi, Kapadia, and Madan (2000).

but only have one OTM put so that  $Z = 5$ , a bias of roughly 0.1 is introduced. If, however, we have two OTM puts, so that  $Z = 10$ , then the bias is essentially zero. We repeat the experiment for the case where we have more data on OTM puts than calls, and perform the integration over the domain  $[10, S(t) + Z]$ . The magnitude of the bias in this case is the same, but is of opposite sign. To mitigate this bias, we use the largest range of strike prices at each time such that the domain of integration is symmetric.

We construct the skew using data from the Berkeley Options Data Base from April 7, 1986, to December 31, 1996. The starting date of our sample corresponds to the date that S&P 500 index options became European-style. We have options data on 1,421 unique firms during the 11-year sample period. The number of firms with listed options in our database increased dramatically over our sample period, from 183 in 1986 to 1,015 in 1996.

We use the midpoint of the last bid-ask quote for each contract on each day. These observations are less than perfectly contemporaneous. Each record contains a concurrent observation of the underlying stock price, however, which allows us to synchronize the option prices. This is accomplished by applying a multiplicative adjustment factor to the strike price, option price, and underlying stock price.<sup>1</sup>

Using the adjusted prices, we compute the BKM measure of the skew each day for the two different maturities that are greater than one week but closest to 22 trading days. To estimate the integrals in equations (2), (3), and (4), we make

<sup>1</sup>To compute the adjustment factors, we establish a "target" closing price by calculating the average reported stock price across all options for that underlying stock that day. The adjustment factor for a particular option quote is simply the ratio of the target price to the reported stock price.

FIGURE 2  
Plot of Skew vs. Half-Width of Integration Domain

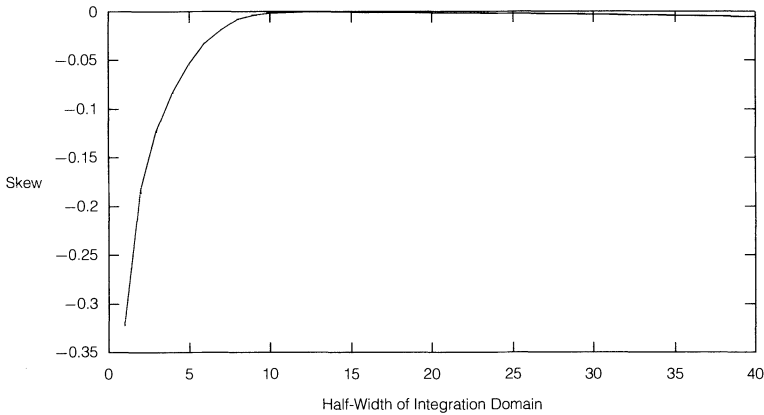


Figure 2 shows the bias that can be introduced into the skew measure when the domain of integration is finite. The domain is symmetric about the current stock price. One hundred Black-Scholes option prices are computed using domain half-widths of \$1 to \$40 centered at \$50. The options have one month to maturity, the underlying stock is \$50, the volatility is 20% per annum, and the risk-free rate is 7%. The risk-neutral skew is then computed from these prices using the method outlined in Bakshi, Kapadia, and Madan (2000).

certain that we have at least two calls and two puts for each maturity. Since the skew may vary with time to maturity, we standardize it for a hypothetical option with 22 trading days to maturity using linear interpolation/extrapolation. We then average the daily measures of the skew to obtain a weekly measure.

### III. Explanatory Variables

This section outlines both our motivation for the inclusion of the explanatory variables and their construction. We construct the independent variables using data from the Berkeley Options Database, CRSP, and COMPUSTAT from April 7, 1986, to December 31, 1996. Since the ticker nomenclature differs between the Chicago Board Options Exchange (CBOE) and the corresponding stock exchange, the Berkeley Options Data Base was merged with CRSP and COMPUSTAT, using a ticker cross-reference provided by the CBOE. To guard against the possibility of erroneous matches, we verified our procedure by requiring that the closing price for the underlying stock from CRSP be within 3% of the closing price of the underlying stock from the Berkeley Options Data Base.

#### A. Implied Volatility

We use the implied volatility of individual stock options for two purposes: to measure the volatility of a firm's stock return and to compare our results using skewness to other studies that have used the slope of the implied volatility curve.

To compute the implied volatility of a hypothetical at-the-money option, the implied volatility for the in-the-money put and the out-of-the-money call whose

strike prices are closest to the stock price are computed and averaged together. This represents the implied volatility for the strike price just above the current stock price. Taking the average helps to reduce noise as well as mitigate any measurement error in the risk-free rate. Likewise, the implied volatility for the out-of-the-money put and the in-the-money call whose strike prices are closest to the stock price are computed and averaged together. The implied volatility of a hypothetical at-the-money option is found by interpolating these two values.

Similar to the computation of the skew, this procedure is done once for the two maturities greater than one week but closest to 22 trading days. Linear interpolation/extrapolation is then used with these two at-the-money implied volatilities to find the implied volatility of an at-the-money option that matures in 22 trading days. The implied volatilities are computed using a 100-step binomial tree, modified to account for multiple discrete dividends and early exercise. Implied volatilities are calculated using bid-ask quote midpoints for the option prices, T-bill rates, and realized dividend data as reported in CRSP. Because the quoted prices of the eight options may not have been observed concurrently, we do not use the closing stock price in our implied volatility calculation, but rather use the stock price reported in the Berkeley Options Data Base that is synchronous with the option quote.

## B. Volume

The importance of liquidity costs in option valuation has long been acknowledged. Simulations conducted by Figlewski (1989) have illustrated the difficulty of implementing dynamic arbitrage strategies. This suggests that option prices are not determined solely by arbitrage but are free to fluctuate within reasonably wide bands. The width of the arbitrage bounds on option prices is determined by the cost of implementing replicating strategies, with tighter bands for stocks with lower transaction costs.

As a proxy for the liquidity cost for each stock, we compute the average daily trading volume in the underlying stock for each week from CRSP. We chose trading volume as our proxy for liquidity since we believe it to be a fairly good measure of the ease of constructing replicating portfolios. We also use average daily turnover as an alternative proxy of liquidity to investigate the robustness of our results. Average daily turnover is defined as trading volume divided by shares outstanding for the week.

## C. Systematic Risk

In the absence of strict arbitrage pricing, one approach is to assume options are priced according to an equilibrium model. Although it is well known that Black-Scholes prices may be supported in equilibrium given normally distributed returns and a representative agent with a constant relative risk aversion, the same is not true for other utility functions, such as those characterized by decreasing relative risk aversion (see, for example, Franke, Stapleton, and Subrahmanyam (1999)). For this reason, Black-Scholes may underprice the insurance premium implicit in state prices corresponding to low index levels. We would expect this phenomenon to manifest itself most clearly in the market for S&P 500



index options. To the extent that this phenomenon is also manifest in individual stock options, we would expect the skew of the risk-neutral density of individual stocks to be more negative in periods when the S&P 500 index option skew is more negative. Also, we would expect the effect to be more pronounced in options with more risk exposure or in options that might best be used to hedge market risk. Thus, we test the hypothesis that skewness is more negative for firms with more market risk, measured by beta.

The beta for stock  $i$  at time  $t$ ,  $BETA_{i,t}$ , is calculated by regressing the daily returns for stock  $i$  on the daily return for the S&P 500 index from day  $t - 200$  to day  $t$ . We also calculate the correlation stock  $i$  at time  $t$  with the S&P 500 index using the same 200 days of data.

#### D. Leverage

As the value of a levered firm declines, the debt-to-equity ratio increases and equity returns may become more volatile. Geske (1979) and Toft and Prucyk (1997) have derived pricing models that assume proportional, constant variance processes for the firm's assets but explicitly account for the impact of risky debt on the dynamics of the firm's equity. These models are based on the notion that return volatility is greater at lower stock price levels, thus implying that out-of-the-money puts have higher implied volatilities than out-of-the-money calls. A priori, we do not believe leverage to be the driving force behind the violations of Black-Scholes, given that similar violations are observed for options on unlevered firms and for currency options where the leverage argument does not apply. Furthermore, BKM (2000) show that the leverage effect implies that the skewness of the risk-neutral density for individual equities should be more negative than that of the index. They find that the opposite is true. Since the effect of leverage is ambiguous, we directly test its relation to the risk-neutral skewness.

We define the leverage ratio for firm  $i$  at time  $t$ ,  $LEV_{i,t}$ , as the ratio of the sum of long-term debt and par value of preferred stock to the sum of long-term debt, preferred stock, and the market value of equity. Book values are used for the value of the long-term debt.

#### E. Trading Pressure

In the presence of transaction costs, option prices may also reflect a liquidity premium that is sensitive to the direction of "trading pressure" from public order flow. One possible explanation for negative skewness in the risk-neutral density is that demand for out-of-the-money puts drives up the prices of low strike price options. Insofar as transaction costs allow option prices to be influenced by investor sentiment, one might predict more skewness in periods when the market is more pessimistic. The ratio of put-to-call trading volume is commonly believed to be a sentiment index, with more put volume indicating pessimism. We wish to test whether the skew is more negative in periods of high put trading.

To investigate the extent to which the risk-neutral skewness may be influenced by trading pressure, we use the ratio of average daily put volume to average daily call volume for each week. Since a trade can be either to open or close

a contract, volume can be a misleading proxy for trading pressure. To address this, we test the robustness of our results using the ratio of average daily put open interest to average daily call open interest for each week. Data on open interest from November 1991 to December 1996 for each option contract were provided to us by an option trading company that is a member of the CBOE and several other option exchanges.

## IV. Sample Properties

### A. Univariate Properties

Figure 3 illustrates how the skews of the risk-neutral density for individual stocks have varied over time. The graph shows that the median skew across firms as a function of time is, on average, negative.

FIGURE 3  
Median Stock Skew

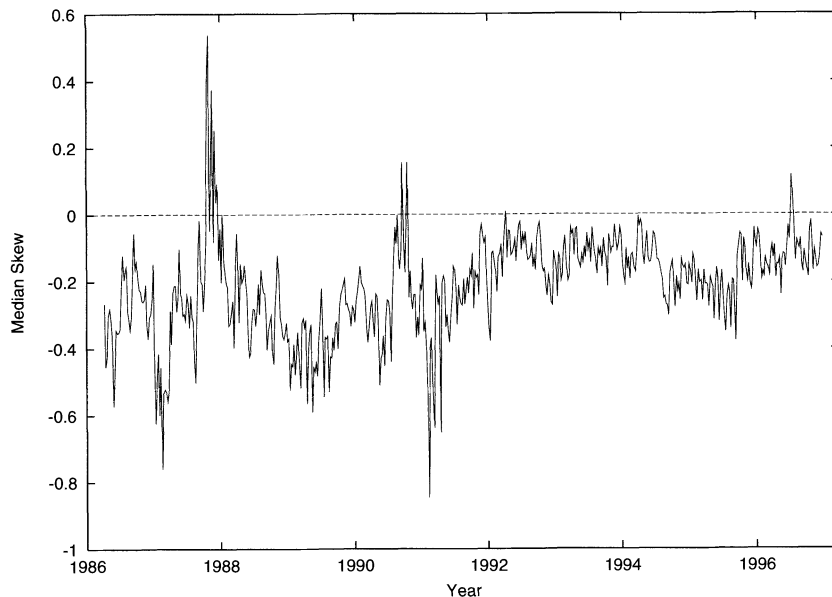


Figure 3 shows the time-series of the median value of the skew of the risk-neutral density for individual stock options trading on the CBOE, observed weekly from April 7, 1986, through December 31, 1996. The skew for each firm was computed using the method in Bakshi, Kapadia, and Madan (2000), which is summarized in equations (1) to (4). Daily observations of the skew are aggregated to a weekly observation as discussed in Section II. The data used in the computations are from the Berkeley Options Data Base that contains intraday option prices for 1,421 firms.

Comparing the skews on S&P 500 index options with that of individual stock options, we observe that the skew for index options tends to be much more negative than the typical skew for individual stock options. The mean skew for the S&P 500 index options in the sample is approximately  $-1.6$ , as compared to  $-0.24$  for individual stock options. In fact, the skew of an individual stock is greater than the skew of the index 92% of the time, similar to the result reported by

BKM (2000). Note that while the skew for individual stocks remained at roughly  $-0.2$  from 1986 to 1996, the average skew of the S&P 500 index options more than tripled in that period, from  $-0.5$  in 1986 to  $-1.6$  in 1996. Furthermore, as observed by Rubinstein (1994), they appear to have become more negative shortly after the crash of October 1987.

Table 1 presents univariate summary statistics for the variables. On average, the skew of the risk-neutral density for individual stocks (SKEW) is negative, but the standard deviation is large, even though we attempt to reduce the noise by averaging the daily skew across each week. As indicated in the table, the average implied volatility is about 46%, which represents the implied volatility of an at-the-money option with 22 trading days to maturity. The mean leverage ratio is 0.27, indicating that the average firm in our sample is financed with roughly three-quarters equity and one-quarter debt and preferred stock. The average firm size (MVE) in our sample is  $\exp(14.42) * 1000 = \$1.83$  billion and, as we would expect, the average beta with the S&P 500 is 1.0. The average daily volume is 436,300 shares, and the average put-to-call volume ratio of 1.45 indicates that roughly three put contracts trade for every two calls. The large standard error of this ratio is driven primarily by lower volume options, where a small change in option volume might constitute a large change in the put-to-call ratio. The mean put-to-call open interest of 0.6 indicates that there are roughly three put contracts open for every five calls.

TABLE 1  
Univariate Statistics for Dependent and Independent Variables

Variable	Mean	First Quartile Breakpoint	Median	Third Quartile Breakpoint	Standard Deviation
SKEW	-0.237	-0.418	-0.192	-0.005	0.479
VOLAT	0.455	0.314	0.409	0.566	0.192
LEVERAGE	0.265	0.054	0.194	0.409	0.254
SIZE	14.42	13.35	14.39	15.43	1.46
BETA	1.020	0.693	0.966	1.284	0.562
VOLUME	436,300	105,280	231,059	514,580	657,256
PUTCALLVL	1.445	0.102	0.295	0.735	12.4
PUTCALLOI	0.619	0.221	0.390	0.618	4.20
SKEW500	-1.622	-2.055	-1.508	-1.102	0.909

Table 1 contains the univariate properties of the firm-specific dependent and independent variables that are used in subsequent regressions. The sample consists of 129,931 observations, constituting weekly observations from April 1986 through December 1996 for the cross section of stock options trading on the Chicago Board Option Exchange. We have options data on 1,421 unique firms during the 11-year sample period. The number of firms with listed options in our database increased over our sample period, from 183 in 1986 to 1,015 in 1996. SKEW is the skew of the risk-neutral density implied by the option prices. VOLAT is the implied volatility of an at-the-money option, LEVERAGE is the firm's leverage ratio, SIZE is the natural log of the firm's market value of equity, in thousands of dollars, BETA is the firm's beta with the S&P 500, VOLUME is the average weekly volume (in shares), PUTCALLVL is the ratio of put volume to call volume for individual options on the firm's stock, PUTCALLOI is the ratio of put open interest to call open interest for individual options on the firm's stock, and SKEW500, the skew of the risk-neutral density implied by S&P 500 index options during week  $t$ .

Table 2 reports the average value of the skew as a function of several independent variables. The data are first sorted into quintiles by the variable of interest, such as size, leverage, etc., and then the average value of the skew within each quintile is reported in the table. Larger firms and firms that have larger vol-

ume have more negative skews. We cannot draw too many conclusions from this because of the collinearity between size and volume. Firms that have low implied volatilities tend to have more negative skews, but these are also the larger, more established firms. Hence, we cannot be sure whether or not it is a size effect. No monotonic relationship appears to exist for skew as a function of leverage and skew as a function of beta.

TABLE 2  
Average Value of the Skew as a Function of Independent Variables

Quintile	VOLAT	LEVERAGE	SIZE	BETA	VOLUME	PUTCALLVL
Smallest	-0.496	-0.156	-0.002	-0.210	-0.177	-0.237
Second	-0.319	-0.267	-0.157	-0.267	-0.197	-0.277
Third	-0.232	-0.267	-0.248	-0.272	-0.227	-0.259
Fourth	-0.156	-0.242	-0.327	-0.239	-0.263	-0.230
Largest	0.058	-0.221	-0.289	-0.203	-0.244	-0.180

Table 2 contains the average value of the skew as a function of the independent variables. The data are sorted from lowest to highest based on the variables VOLAT, LEVERAGE, etc. Based on these sorts, quintiles are formed using the sort variable (VOLAT, LEVERAGE, etc.) and the mean value of the SKEW in each quintile is computed and reported in this table. The sample contains weekly observations from April 1986 through December 1996 for the cross section of stock options trading on the Chicago Board Option Exchange. VOLAT is the implied volatility of an at-the-money option, LEVERAGE is the firm's leverage ratio, SIZE is the natural log of the firm's market value of equity, in thousands of dollars, BETA is the firm's beta with the S&P 500, VOLUME is the average weekly volume (in shares), and PUTCALLVL is the ratio of put volume to call volume for individual options on the firm's stock.

## B. Correlations

Table 3 reports the correlation coefficients between the variables used in our analysis. The correlations are consistent with the statistics on skew presented in Table 2. The level of the implied volatility is negatively correlated with leverage and size, implying that large firms with more leverage have lower volatility. Volatility is also positively correlated with the firm's beta, and there is a positive correlation between size and volume. These correlations indicate a potential multicollinearity problem in our regression analysis.

## V. Results

### A. Cross-Sectional Analysis

As a first step toward understanding how firm-specific factors influence the firm's risk-neutral skew we estimate weekly cross-sectional regressions of the form,

$$(5) \text{SKEW}_i = a_0 + a_1 \text{VOLAT}_i + a_2 \text{LEVERAGE}_i + a_3 \text{SIZE}_i + a_4 \text{BETA}_i + a_5 \text{VOLUME}_i + a_6 \text{PUTCALL}_i + \epsilon_i,$$

where  $\text{SKEW}_i$  is the skew of the risk-neutral density for the firm for firm  $i$ ,  $\text{VOLAT}_i$  is the implied volatility of an at-the-money option,  $\text{LEVERAGE}_i$  is the leverage ratio,  $\text{SIZE}_i$  is the market value of equity,  $\text{BETA}_i$  is the beta of the stock's

TABLE 3  
Correlations between Variables

	SKEW	VOLAT	LEV	SIZE	BETA	VOL	PUTCALL VOLUME	PUTCALL OPENINT
VOLAT	0.3725	.						
LEVERAGE	-0.0174	-0.2550	.					
SIZE	-0.3110	-0.6852	0.0637	.				
BETA	0.0585	0.2749	-0.1588	-0.0300	.			
VOLUME	-0.0921	-0.0904	-0.0793	0.5941	0.2597	.		
PUTCALLVL	-0.0057	0.0186	-0.0050	0.0083	0.0139	0.0410	.	
PUTCALLOI	-0.0259	-0.0210	0.0041	0.0205	-0.0030	-0.0033	0.0053	.
SKEW500	0.0074	-0.0949	0.0665	0.1240	-0.0448	-0.0191	-0.0045	0.3305

Table 3 contains the correlation coefficients between the firm-specific variables that are used in subsequent regressions. The sample consists of 90,106 observations, constituting weekly observations from April 1986 through December 1996 for the cross section of stock options trading on the Chicago Board Option Exchange. SKEW is the skew of the risk-neutral density implied by the option prices; VOLAT is the implied volatility of an at-the-money option; LEVERAGE is the firm's leverage ratio; SIZE is the natural log of the firm's market value of equity, in thousands of dollars; BETA is the beta of the firm's returns with the S&P 500; VOLUME is the natural logarithm of the average weekly volume (in shares); PUTCALLVL is the ratio of put volume to call volume for individual options on the firm's stock; and PUTCALLOI is the ratio of put open interest to call open interest for individual options on the firm's stock; SKEW500, the skew of the risk-neutral density implied by S&P 500 index options during week  $t$ .

returns with the S&P 500,  $VOLUME_i$  proxies for the liquidity of the underlying stock, and  $PUTCALL_i$  is the ratio of put-to-call volume. The construction of the variables is explained in Sections II and III.

To begin our analysis we use a Fama-Macbeth (1973) type of approach, estimating the model once for each week in our sample period. This yields 545 estimates of each coefficient. The time-series averages of the estimated coefficients in each of the weekly cross-sectional regressions are reported in panel A of Table 4 and the  $t$ -statistics correspond to a null hypothesis that the time-series mean is zero.

Examining the results, we come to the following conclusions. First, the coefficient on the level of implied volatility, VOLAT, is positive and significant, indicating that high volatility firms have less negative skews. Second, contrary to the results of Toft and Prucyk (1997), we find that the coefficient on LEVERAGE is positive and significant. We address this issue in more detail in Section VII.

Third, the coefficient on BETA is negative and statistically significant, meaning that firms with more systematic risk have a more negatively skewed risk-neutral density. The fact that market risk helps explain the cross-sectional variation in individual firm skews suggests that individual stock options ought to be priced using models that incorporate market risk.

Fourth, the significantly negative coefficient on SIZE indicates that large firms have more negative skewness than do small firms. We include size to control for any omitted risk factors, as suggested by Fama and French (1992) and others, but do not have a strong prior for the sign of coefficient. However, if smaller firms are riskier than large firms, we would expect that smaller firms would have more negative skews than large firms, which is the opposite of what we find.

Fifth, the positive coefficient on VOLUME indicates that stocks with higher volume have more positive skews. This is consistent with two explanations. First, it could be the case that trading volume is a proxy for differences in the physical

distribution of stock returns, so that high volume stocks have less skewness under the physical measure, and this carries over to the risk-neutral measure. Second, it could be that risk-neutral densities are close to symmetric in frictionless markets, where options are priced by arbitrage, but that illiquid markets allow option prices to be affected by investor preferences or trading pressure in a way that leads to negative skewness in the implied risk-neutral density.

Last, the coefficient on the put/call ratio is positive. However, both the economic and statistical significance of this result is marginal and in robustness checks performed in Section VI the statistical significance vanishes altogether. Recall that we include the put/call ratio as a proxy for investor sentiment or trading pressure, the hypothesis being that the risk-neutral skew may be influenced by

TABLE 4  
Determinants of the Risk-Neutral Skew: Cross-Sectional Regressions

<i>Panel A. All Weeks</i>									
	Mean Coefficient		% t-Stats		n/p				
VOLAT	0.620 (18.4)		0/43						
LEVERAGE	0.063 (3.7)		3/12						
SIZE	-0.066 (-9.9)		30/1						
BETA	-0.038 (-2.4)		15/4						
VOLUME	0.030 (6.9)		2/15						
PUTCALLVL	0.011 (3.0)		3/4						
Mean $R^2$ (%)	17.5								
<i>Panel B. Quartiles Sorted by Index Skewness</i>									
	(Low Idx Skew)		Quartile 2		Quartile 3		(High Idx Skew)		
	Quartile 1						Quartile 4		
	Mean Coeff	% t-Stat n/p	Mean Coeff	% t-Stat n/p	Mean Coeff	% t-Stat n/p	Mean Coeff	% t-Stat n/p	
VOLAT	0.542 (17.9)	0/51	0.619 (13.0)	0/40	0.666 (16.9)	0/43	0.670 (14.6)	0/38	
LEVERAGE	0.082 (6.7)	1/21	0.058 (3.5)	2/9	0.056 (3.6)	1/11	0.051 (2.8)	0/7	
SIZE	-0.069 (-8.6)	40/1	-0.076 (-10.1)	36/1	-0.054 (-7.9)	24/0	-0.066 (-10.0)	19/3	
BETA	0.044 (-4.5)	17/5	-0.085 (-7.0)	23/1	-0.031 (-2.5)	15/3	0.004 (0.3)	6/6	
VOLUME	0.031 (6.5)	3/21	0.029 (5.2)	1/13	0.026 (4.2)	3/13	0.034 (6.1)	1/12	
PUTCALLVL	0.008 (3.3)	3/3	0.003 (0.5)	3/4	0.013 (2.9)	3/3	0.020 (2.2)	4/7	
Mean Index Skew	-2.447		-1.464		-1.061		-0.404		
Mean Stock Skew	-0.238		-0.277		-0.246		-0.252		
Mean $R^2$ (%)	18.5		18.0		17.3		16.5		

(continued on next page)

TABLE 4 (continued)  
 Determinants of the Risk-Neutral Skew: Cross-Sectional Regressions

Panel C. Quartiles Sorted by Index Volatility

	(Low Idx Volat) Quartile 1		Quartile 2		Quartile 3		(High Idx Volat) Quartile 4	
	Mean Coeff	% <i>t</i> -Stat <i>n/p</i>	Mean Coeff	% <i>t</i> -Stat <i>n/p</i>	Mean Coeff	% <i>t</i> -Stat <i>n/p</i>	Mean Coeff	% <i>t</i> -Stat <i>n/p</i>
VOLAT	0.511 (15.7)	0/50	0.615 (16.2)	0/37	0.636 (12.7)	0/41	0.719 (17.1)	0/45
LEVERAGE	0.021 (1.8)	2/10	0.053 (3.5)	1/12	0.098 (5.5)	5/17	0.075 (4.4)	1/10
SIZE	-0.056 (-8.5)	35/1	-0.070 (-8.9)	26/1	-0.079 (-11.0)	35/1	-0.059 (-8.2)	23/2
BETA	-0.037 (-6.0)	13/1	-0.036 (-3.1)	13/3	-0.042 (-2.7)	19/7	-0.039 (-2.3)	16/4
VOLUME	0.024 (5.8)	2/17	0.037 (6.6)	1/15	0.038 (6.5)	1/14	0.021 (3.4)	2/13
PUTCALLVL	0.001 (0.3)	3/3	0.009 (1.8)	4/1	0.012 (1.6)	3/4	0.022 (2.6)	4/10
Mean Index Vol.	0.137		0.169		0.203		0.285	
Mean Stock Skew	-0.206		-0.266		-0.314		-0.227	
Mean $R^2$ (%)	16.9		16.8		18.4		18.0	

Table 4 contains the time-series averages of the coefficients from weekly cross-sectional regressions of the skew of the risk-neutral density on explanatory variables. The model is

$$\text{SKEW}_i = a_0 + a_1 \text{VOLAT}_i + a_2 \text{LEVERAGE}_i + a_3 \text{SIZE}_i + a_4 \text{BETA}_i + a_5 \text{VOLUME}_i + a_6 \text{PUTCALL}_i + \epsilon_i,$$

where  $i$  indexes the firm. SKEW is the skew of the risk-neutral density for firm  $i$  implied by its option prices. The independent variables are VOLAT, the implied volatility of an at-the-money option; LEVERAGE, the total value of long-term debt plus preferred stock, divided by the sum of long-term debt, preferred stock, and equity; SIZE, the logarithm of the market value of equity; BETA, the beta of firm  $i$  with the S&P 500; VOLUME, the natural logarithm of the average weekly volume (in shares) during week  $t$ ; and PUTCALL, the total trading volume in all put options on firm  $i$  divided by the trading volume in all call options on firm  $i$ . In each panel the time-series  $t$ -statistics are in parentheses and are computed using the time-series of coefficients. The  $t$ -statistics in panel A are corrected for first-order serial correlation. The percentage of  $t$ -statistics from the cross-sectional regressions that are negative and significant at the 5% level ( $n$ ) and positive and significant at the 5% level ( $p$ ) are shown in the column labeled  $n/p$ . Panel A shows the time-series averages for all 545 weeks in the sample period. Panel B shows the time-series averages for quartiles based on the skew of density of the S&P500, and panel C shows the time-series averages for quartiles based on the volatility of the S&P500. There are 136 weekly observations in each quartile in panels B and C.

pessimistic traders driving up prices for “bad” states of the world. This would be consistent with a negative coefficient on the put/call ratio. We find no evidence in support of this explanation.

In addition to firm-specific factors, there may be market-wide factors that affect a firm’s risk-neutral skew. For example, a firm’s skew may have a systematic component that is related to the skew of the market index. To control for market-wide factors in the context of the cross-sectional regressions, we partition the data into quartiles and repeat the cross-sectional analysis. Panel B of Table 4 contains the time-series average of the coefficients where the data are partitioned into quartiles based on the skewness of the S&P 500 index. The mean skew for the S&P 500 index is much more negative than for individual stocks, increasing from a mean of  $-2.447$  for the lowest quartile to a mean of  $-0.404$  for the highest quartile. The skew for individual equities is much less negative than that of the

index, varying from  $-0.24$  to  $-0.28$ , and shows no apparent monotonic relationship to the skew of the index. In Section V.B, we investigate this matter further in a pooled cross-sectional time-series regression framework. The relationship between the firm's skew and the firm-specific variables is, for the most part, identical to those in panel A and do not appear to be driven by the index skew.

In panel C of Table 4, we sort and partition the weekly observations into quartiles by the market volatility. We use the implied volatility of an at-the-money S&P 500 index option to measure market volatility. The mean market volatility is 13.7% (28.5%) per annum for the lowest (highest) quartile. There does appear to be a monotonic relationship between the volatility of the index and the mean stock skew, where the skew of individual stocks becomes more negative as the index volatility increases for the three lowest quartiles. The sign and significance of the estimated coefficients on the firm-specific variables in each of the four quartiles are identical to those in panel A, with the exception of the estimated coefficient on the ratio of put-to-call volume, which is only significant in one of the four quartiles.

## B. Panel Analysis

While the cross-sectional analysis provides a good starting point, it does not allow us to fully examine how market-wide factors that are common to all firms may affect an individual firm's skew. To further examine the relation between the risk-neutral skew, firm-specific variables, and market-wide variables, we estimate the following pooled time-series cross-sectional regression,

$$(6) \quad \text{SKEW}_{i,t} = a_0 + a_1 \text{VOLAT}_{i,t} + a_2 \text{LEVERAGE}_{i,t} + a_3 \text{SIZE}_{i,t} \\ + a_4 \text{BETA}_{i,t} + a_5 \text{VOLUME}_{i,t} + a_6 \text{PUTCALL}_{i,t} \\ + a_7 \text{SKEW500}_t + a_8 \text{VOL500}_t + a_9 \text{SKEW}_{i,t-1} + \epsilon_{i,t},$$

where  $i$  indexes the firm and  $t$  indexes the week of the observation. In addition to the variables included in model (5), we include two market-wide variables: the skew of the S&P 500 ( $\text{SKEW500}_t$ ) and the volatility of the market index, proxied by the implied volatility of an at-the-money S&P 500 index option ( $\text{VOL500}_t$ ).

Parameter estimates for regression equation (6) are reported in the first column of Table 5. The table also reports  $t$ - and  $R^2$ -statistics. The sign and significance of the estimated coefficients for the firm-specific variables are, in general, the same as the time-series average of the cross-sectional coefficients reported in Table 4. The two differences are that the put-to-call volume ratio and the volume of the underlying firm lose significance at even the 10% level in the pooled model. The two market-wide variables, the index skew and volatility, are significant at the 1% level. While there was not an obvious monotonic relationship in Table 4 between the skew of the firm and that of the S&P 500 index, in the pooled regression we find that there is a systematic component of the skew of the individual stock. In other words, the risk-neutral density for individual stock options is more negatively (positively) skewed in time periods when the risk-neutral density for index options is more negatively (positively) skewed.

Consistent with the results reported in panel C of Table 4, we find that the estimated coefficient on  $\text{VOL500}$  is negative and significant, indicating that a firm's



TABLE 5  
Determinants of the Risk-Neutral Skew: Panel Regressions

	Full Model	Drop Size	Drop Volume
Intercept	0.0276 (1.1)	-0.0704 (-3.7)	0.0279 (1.1)
LEVEL	0.3156 (17.6)	0.3787 (22.1)	0.3117 (18.7)
LEVERAGE	0.0380 (7.6)	0.0430 (8.7)	0.0378 (7.6)
SIZE	-0.0126 (-5.3)		-0.0135 (-8.8)
BETA	-0.0182 (-7.7)	-0.0188 (-7.8)	-0.0185 (-7.7)
VOLUME	-0.0011 (-0.6)	-0.0100 (-8.7)	
PUTCALL	-0.0001 (-1.7)	-0.0001 (-1.6)	-0.0001 (-1.7)
SKEW500	0.0218 (13.8)	0.0209 (13.3)	0.0219 (13.9)
VOL500	-0.1134 (-3.8)	-0.1381 (-4.8)	-0.1121 (-3.8)
SKEW <sub><i>t</i>-1</sub>	0.6332 (34.7)	0.6354 (35.0)	0.6331 (34.7)
R <sup>2</sup> (%)	48	48	48
N	88,901	88,901	88,901

Table 5 contains the results of cross-sectional regressions of the skew of the risk-neutral density on explanatory variables. The *t*-statistics, computed using White's robust standard errors, are shown in the parentheses below coefficient estimates. The models are of the form,

$$\begin{aligned} \text{SKEW}_{i,t} = & a_0 + a_1 \text{VOLAT}_{i,t} + a_2 \text{LEVERAGE}_{i,t} + a_3 \text{SIZE}_{i,t} + a_4 \text{BETA}_{i,t} \\ & + a_5 \text{VOLUME}_{i,t} + a_6 \text{PUTCALL}_{i,t} + a_7 \text{SKEW500}_t + a_8 \text{VOL500}_t \\ & + a_9 \text{SKEW}_{i,t-1} + \epsilon_{i,t}, \end{aligned}$$

where *i* indexes the firm and *t* indexes the week for the period from April 7, 1986, to December 31, 1996. SKEW is the skew of the risk-neutral density implied by the option prices. The independent variables are VOLAT, the implied volatility of an at-the-money option; LEVERAGE, the total value of long-term debt plus preferred stock, divided by the sum of long-term debt, preferred stock, and equity; SIZE, the logarithm of the market value of equity; BETA, the beta of firm *i* with the S&P 500; VOLUME, the natural logarithm of the average weekly volume (in shares) during week *t*; PUTCALL, the total trading volume in all put options on firm *i* divided by the trading volume in all call options on firm *i*; SKEW500, the skew of the risk-neutral density implied by S&P 500 index options during week *t*; and VOL500, the volatility of the S&P500 index.

skewness tends to be more negative during periods of high market volatility. It is somewhat of a puzzle that the skew is more negative when market volatility is high, yet the skew is more positive when the firm's own volatility is high. In the robustness analysis below, however, the coefficient on market volatility loses significance.

Last we include the lagged skew to proxy for any omitted firm-specific variables and capture the persistence in the skew. In Table 5, the coefficient on the lagged skew is positive and significant, indicating that cross-sectional differences in risk-neutral skews are persistent over time.

Recall from Section IV.B that there is a potential multicollinearity problem, particularly with the variables VOLAT, SIZE, and VOLUME. To assess the impact of multicollinearity, we re-estimate the model with SIZE omitted and also

with VOLUME omitted. Parameter estimates for these specifications are reported in columns two and three of Table 5. None of the results change, except that when SIZE is omitted, the coefficient on VOLUME becomes negative and significant. In this specification, VOLUME appears to become a proxy for firm size.

### C. Systematic vs. Firm-Specific Factors

One of our goals is to measure the relative importance of systematic factors vs. firm-specific factors in explaining skews of individual stock options. We use the skew of the S&P 500 index as a proxy for systematic factors affecting the skew. When we estimate the model in (6), we find a positive coefficient on SKEW500, which indicates that systematic factors explain at least some of the time-series variation in skews. Furthermore, the positive coefficient on beta suggests that systematic risk in the underlying stock explains at least some of the cross-sectional variation in the skew.

In addition to our proxy for the systematic factors affecting the skew on individual stocks, we have also included many firm-specific factors such as leverage, firm size, and put-to-call volume. While we have explicitly accounted for some firm-specific factors, there may be others that we have omitted that affect skewness. The lagged value of the dependent variable,  $SKEW_{i,t-1}$ , was included in the regression specified in model (6) to subsume these omitted firm-specific factors.

To isolate the effect of systematic vs. firm-specific factors on the skew, we re-estimate the regression in Table 5 three different ways. As a baseline, we omit both SKEW500 and  $SKEW_{t-1}$  and find that the explanatory power of the model is much lower ( $R^2 = 9.3\%$ ). Next we include only the proxy for the systematic factors that affect the skew, SKEW500, and find only a slight improvement in the explanatory power ( $R^2 = 9.4\%$ ). Last, we exclude the skew for the index and include  $SKEW_{t-1}$ , which should subsume any omitted firm-specific factors. We find a large improvement in the explanatory power ( $R^2 = 44.2\%$ ) when compared to the model that only includes our proxy for systematic factors.

The relationship between individual stock skews and the market skew is discussed extensively by BKM (2000). As these authors note, individual stock skewness may be expressed as a positive weighted combination of the market and idiosyncratic skew. Our results suggest that firm-specific factors are more important than the variation in systematic factors in explaining the variation in the skew on individual stocks. This implies that in seeking an improved model for pricing individual stock options, we should seek to better understand the sources of idiosyncratic risk.

## VI. Robustness Tests

To investigate the robustness of the results presented in the previous section, we perform various tests. Several dimensions are examined, including bias due to correlated residuals, alternative variable definitions, liquidity issues, and differences before and after October 1987. Some of these results are reported in Table 6.

TABLE 6  
Robustness Analysis

	Turnover	Most Liquid	Before 10/87	After 10/87
Intercept	0.0292 (1.2)	-0.1771 (-2.7)	0.1992 (3.0)	0.0902 (3.5)
VOLAT	0.3267 (18.2)	0.4053 (8.1)	0.1742 (3.8)	0.3055 (16.2)
LEVERAGE	0.0359 (7.3)	0.0479 (4.2)	0.0276 (1.8)	0.0373 (7.2)
SIZE	-0.0137 (-8.9)	-0.0058 (-0.9)	-0.0235 (-4.1)	-0.0127 (-5.2)
BETA	-0.0160 (-6.6)	-0.0297 (-5.7)	0.0162 (1.1)	-0.0200 (-8.2)
VOLUME		0.0039 (0.7)	0.0109 (2.0)	-0.0018 (-1.0)
TURNOVER	-0.8386 (-3.6)			
PUTCALL	-0.0001 (-1.6)	0.0000 (-0.3)	0.0096 (1.4)	-0.0002 (-1.8)
SKEW500	0.0215 (13.6)	0.0352 (9.0)	0.2120 (19.9)	0.0256 (15.6)
VOL500	-0.1194 (-4.1)	-0.1093 (-1.8)	-0.2168 (-2.3)	-0.3356 (-11.3)
SKEW <sub><i>i,t-1</i></sub>	0.6325 (34.7)	0.6010 (13.0)	0.7112 (66.5)	0.6255 (31.7)
<i>R</i> <sup>2</sup> (%)	48	42	60	47
<i>N</i>	88,901	25,281	6,976	81,541

Table 6 contains the results of cross-sectional regressions of the skew of the risk-neutral density on explanatory variables. The *t*-statistics, computed using White's robust standard errors, are shown in the parentheses below coefficient estimates. The models are of the form,

$$\begin{aligned} \text{SKEW}_{i,t} = & a_0 + a_1 \text{VOLAT}_{i,t} + a_2 \text{LEVERAGE}_{i,t} + a_3 \text{SIZE}_{i,t} + a_4 \text{BETA}_{i,t} \\ & + a_5 \text{VOLUME}_{i,t} + a_6 \text{PUTCALL}_{i,t} + a_7 \text{SKEW500}_t + a_8 \text{VOL500}_t \\ & + a_9 \text{SKEW}_{i,t-1} + \epsilon_{i,t}, \end{aligned}$$

where *i* indexes the firm and *t* indexes the week. SKEW is the skew of the risk-neutral density implied by the option prices. The independent variables are VOLAT, the implied volatility of an at-the-money option; LEVERAGE, the total value of long-term debt plus preferred stock, divided by the sum of long-term debt, preferred stock, and equity; SIZE, the logarithm of the market value of equity; BETA, the beta of firm *i* with the S&P 500; VOLUME, the natural logarithm of the average weekly volume (in shares) during week *t*; TURNOVER, the average daily volume divided by shares outstanding; PUTCALL, the total trading volume in all put options on firm *i* divided by the trading volume in all call options on firm *i*; SKEW500, the skew of the risk-neutral density implied by S&P 500 index options during week *t*; and VOL500, the volatility of the S&P500 index.

First, the *t*-statistics in Table 5 may be biased upward due to serial correlation in the residuals. Serial correlation may result from the fact that our observations are weekly while the leverage ratio is updated only once each quarter and the measure of the skew is for an option with 22 days to maturity. As a robustness check, we estimated the full model in Table 5 using non-overlapping 22-day intervals. Specifically, we estimated the full model in Table 5 three times using every third week (once starting with week one, once starting with week two, and once starting with week three). We find that the economic and statistical significance of our results is the same as in Table 5.

Second, because we found that volume was only marginally significant and because some researchers argue that turnover is a more meaningful measure of liquidity than volume, we performed our analysis using turnover instead of trading volume. These results are reported in the first column of Table 6. In this specification, turnover is not significant, but the signs of all the other independent variables are the same as before.

Third, we are concerned that an excessive amount of noise may be introduced by observations on options with light trading volume, particularly in our measure of the put-to-call ratio. For example, if only two call contracts have traded, then a small change in put volume from one contract to four contracts would cause a large change in the put-to-call ratio from 0.5 to 2.0. To address this, we re-estimate the model using only the most active observations, defined as those weeks when the sum of put and call contract volume places the observation in the upper quartile of all observations. These results are in the second column of Table 6. In this sample, the coefficient on SIZE is no longer significant, and all the other results are basically the same as those found in Table 5.

Fourth, because Rubinstein (1994) reports a significant change in the slope of the implied volatility curve (or smile) on index options around the time of the market crash in October 1987, we are concerned that some type of structural change may have occurred at that point. The last two columns of Table 6 contain the results from estimating the model before and after October 1987. Two regressions are estimated, the first using observations from April 7, 1986, to October 1, 1987, and the second using observations from November 1, 1987, to December 31, 1996. Examining the last two columns of Table 6, we find that the coefficient on VOLAT, SIZE, SKEW500, VOL500, and the lagged skew are significant in both subsamples, while LEVERAGE and BETA are only significant in the post-October 1987 sample. The variables that are significant in these subsamples have the same sign as in the full sample. The fact that BETA is only significant in the post-crash period suggests that market risk became more important to option holders after the crash.

We perform several more robustness tests that are not formally reported here. Since volume may not adequately capture the number of open contracts at a given time, we re-estimate the regression substituting the ratio of put open interest to call open interest for the variable PUTCALL. The signs and significance of the coefficients remain unchanged in both the full sample and in the sample of liquid stocks. Trading pressure does not appear to explain the skew of the risk-neutral density.

In addition, we re-estimate the model replacing beta with the estimated correlation between the stock and the S&P 500 index. Although highly correlated with beta, this measure more accurately represents the ease with which the stock option may be used as a substitute vehicle for hedging market risk. The correlations at time  $t$  are computed using daily returns from day  $t$  to day  $t - 200$ . The coefficient is negative and significant, and the sign and significance of the other independent variables from this specification remain unchanged.

## VII. Further Investigation on the Leverage Effect

In Section V.A, we found that higher leverage leads to more right-skewness in the risk-neutral density. This is puzzling, since it is contrary to what theory would predict and also contrary to the results reported by Toft and Prucyk (1997). In this section, we investigate the relationship between the level of implied volatility and leverage and show that the construction of the implied volatility curve measure used by those authors can induce spurious results.

The univariate results presented in Section V.A above indicate a negative correlation between leverage and the level of implied volatility. To investigate this further, we regress the implied volatility of at-the-money options on leverage, controlling for various combinations of firm-specific and market-wide factors. The results confirm that leverage is negatively related to the level of implied volatility. At first, this may seem odd, since theory suggests that if a firm adds more debt to its capital structure, the volatility of equity will increase. The negative relationship between volatility and leverage is a cross-sectional phenomenon—low volatility firms are able to take on more debt in their capital structure.

The measure used by Toft and Prucyk (1997) was constructed by dividing a linear approximation of the slope of the implied volatility curve by the implied volatility of an at-the-money option. Because volatility is related to leverage, the Toft-Prucyk result that higher leverage leads to more left-skewness may be a spurious artifact of their skewness measure. To investigate this, we re-estimate the regressions in Table 5 using the Toft-Prucyk measure of the skew as the dependent variable and we are able to replicate their result that the coefficient on LEVERAGE is significant and negative. Then, we use a linear approximation to the slope of the implied volatility curve *not* divided by the implied volatility and find leverage to be positively related to the slope of the volatility curve.<sup>2</sup> Hence, it appears that the results obtained by Toft and Prucyk are being driven by the negative relationship between implied volatility and leverage.

## VIII. Conclusion

In this paper, we investigate the skewness of the risk-neutral density implied by option prices for individual stock options. Our goal is to better understand the dominant economic factors influencing the prices of individual stock options. Our results have implications that may guide future theoretical research in option pricing.

First, we find evidence that the market risk seems to matter in pricing individual stock options. The risk-neutral density implied by option prices tends to be more negatively skewed for stocks with higher betas, in periods of higher market volatility, and in periods when the implied density from index options is more negatively skewed. That market risk matters implies that individual stock options values cannot be determined solely by no-arbitrage arguments, and underscores the importance of theoretical work on equilibrium option pricing models. It may be worth further exploring models that price individual stock options based on the joint dynamics of the underlying stock and the market portfolio.

<sup>2</sup>These results are available on request.

Second, we find evidence that firm-specific factors are important in explaining the variation in the skew for individual firms. Liquidity (proxied by underlying trading volume) and firm size both help explain cross-sectional variation in the skew. In addition, evidence from panel data indicates the presence of an unexplained firm-specific component more important than all of our other explanatory variables put together. Further firm-specific analysis may reveal additional important factors that help explain stock option prices. Additional factors might be uncovered through a careful analysis of historical returns data, or perhaps by incorporating relevant forward-looking information about the firm's real investment portfolio.

Third, we find no robust cross-sectional relationship between risk-neutral skewness and the underlying stock's leverage ratio. The implication is that it may not be worthwhile pursuing the approach to option pricing that explicitly incorporates the firm's capital structure, as in the models of Geske (1979) and Toft and Prucyk (1997). We do not recommend that this line of research be dismissed entirely, as such models may still be useful for pricing options on stocks that are close to bankruptcy. But for the typical stock, leverage does not appear to be an important factor. This conclusion is consistent with and complements those reported by Figlewski and Wang (2000).

Other interesting issues remain to be addressed. Alternative stochastic processes such as stochastic volatility, GARCH, Poisson, regime-switching, variance Gamma, Levy, and others have been advanced to explain the observed biases of the Black-Scholes model. Most of these models appear to fit index option prices better than does Black-Scholes, but it is difficult to differentiate between them using only data from index options. Individual stock options provide a fertile ground for comparing these models because option prices are available on a relatively large cross section of stocks. It would be interesting to measure the extent to which the residual variation in risk-neutral skewness can be explained by historical parameter estimates from these alternative models.

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